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ON COMPUTING A FLOW OF A BINARY GAS MIXTURE*

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Equations obtained from the gas kinetic equations using a method given in /2/, were used in /1/ to solve the problem of binary gas mixture flow through a plane slit and a cylindrical pipe with adhesion (velocity at the channel wall equal to zero). The present paper considers the solution of equations /1/ for a more general, real boundary condition corresponding to the interaction between the particles and the wall, with arbitrary values of the coefficients of accommodation of tangential impulse ε_i. This leads to appearance of additional differences in the component velocities caused by the slippage on the walls.

Following /1/ we first consider a flow along the x-axis between to planes at the distance of $y=\pm h$ apart. The initial system of hydrodynamic equations has the form /1/ (D_{12} and μ_i are the coefficients of mutual diffusion and viscosity of the separate components)

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{d}{\mu_1}(u_2-u_1)=\frac{1}{\mu_1}\frac{\partial P_1}{\partial x} \tag{1}$$
$$\frac{\partial^2 u_2}{\partial y^2}+\frac{d}{\mu_2}(u_1-u_2)=\frac{1}{\mu_2}\frac{\partial P_2}{\partial x}$$
$$(d=P_1P_2/(PD_{12}),\ P=P_1+P_2)$$

We can write the solution of this system, after averaging over the channel crossection, in the form ($u_i(h)$ denote certain values of the velocities at the channel wall, and should be specified)

$$\langle u_1\rangle=-\frac{h^2}{3(\mu_1+\mu_2)}\frac{dP}{dx}+u_1(h)-\frac{1}{k_0^2}\left[\omega_1+\frac{d}{\mu_1}(u_1(h)-u_2(h))\right]\times\left[1-\frac{\operatorname{th}(k_0h)}{k_0h}\right] \tag{2}$$

$$\omega_i=\frac{1}{\mu_i}\frac{dP_i}{dx}-\frac{1}{\mu_1+\mu_2}\frac{dP}{dx},\quad k_0^2=d\frac{\mu_1+\mu_2}{\mu_1\mu_2}$$

($\langle u_2\rangle$ is obtained by interchanging the subscripts 1 and 2). When $u_i(h)=0$, the solution (2) becomes identical to the result of /1/.

Next we determine the velocity $u_i(h)$, remaining within the accuracy of the equations (1). We do this by considering the gas kinetic equations /2/ from which (1) were derived, and the form of the distribution function near the wall. When the particles are reflected from the walls according to the Maxwell diffusive-specular reflection law, then the distribution functions of the incident (f_i^-) and reflected (f_i^+) particles near the wall are connected by the relation

$$f_i^+(v^x,v^y)=(1-\varepsilon_i)f_i^-(v^x,-v^y)+\varepsilon_i f_i^\circ \tag{3}$$
$$f_i^\circ=n_i\left(\frac{\beta_i}{\pi}\right)^{3/2}\exp(-\beta_i v^2),\qquad \beta_i=\frac{m_i}{2kT} \tag{4}$$

with $f_i^+=0$ when $v^y<0$, $f_i^-=0$ and $v^y>0$. On the other hand, it can be shown (see /2/) that the formulas (1) were actually derived with help of a distribution function of the form

$$f_i=f_i^{(0)}\left[1-\frac{2\mu_i}{P_i}\beta_i(v^x-u_i)v^y\frac{\partial u_i}{\partial y}\right] \tag{5}$$

$$f_i^{(0)}=n_i\left(\frac{\beta_i}{\pi}\right)^{3/2}\exp\{-\beta_i[(v^x-u_i)^2+(v^y)^2+(v^z)^2]\} \tag{6}$$

where we have the following relation for the nondiagonal element of the stress tensor:

$$P_i^{xy}=-\mu_i\partial u_i/\partial y \tag{7}$$

Taking now (5) with (6) as f_i^- and using (3), (4), we obtain the following expression for P_i^{xy} near the wall:

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$$P_i^{xy} = -\frac{\varepsilon_i}{2}\left[\mu_i \frac{\partial u_i}{\partial y} + 2P_i\left(\frac{\beta_i}{\pi}\right)^{1/2} u_i\right] \tag{8}$$

Extending (7) to the gaseous layer adjacent to the wall, with thickness of the order of mean free path of the molecule we obtain, from (8) and (7), the boundary condition of slippage for a binary gas mixture described by the system (1)

$$u_i(\mp h) = \pm \frac{b_i \mu_i}{2P_i}\left(\frac{\pi}{\beta_i}\right)^{1/2} \frac{\partial u_i}{\partial y}\bigg|_{y=\mp h}, \qquad b_i = \frac{2-\varepsilon_i}{\varepsilon_i} \tag{9}$$

Solution of (1) together with (9) yields $u_i(h)$. Substituting the latter into (2), we obtain the required solution of the problem of a flow in a plane slit. We shall limit ourselves for brevity to quoting the result for the case $k_0 h \gg 1$ ($k_0 h \sim \mathrm{Kn}^{-1}$ (Kn is Knudsen number). We have, with the accuracy of up to the terms $\sim \mathrm{Kn}^2$,

$$\begin{aligned}\langle u_1\rangle = &-\left[\frac{h^2}{3(\mu_1+\mu_2)} + \frac{2Bh}{1+C}\right]\frac{dP}{dx} - [(\mu_1+\mu_2)(1+C)]^{-1} D_{12} \times \\ &\left\{[\mu_2 + A_1 y_2(\mu_1+\mu_2)]\frac{1}{y_1 y_2}\frac{dy_1}{dx} + \left(\frac{\mu_2}{y_2} - \frac{\mu_1}{y_1}\right)\left[\frac{\mu_2}{\mu_1+\mu_2}(1-C) + \right.\right. \\ &\left.\left. A_1 y_2\right]\frac{1}{P}\frac{dP}{dx} + \mu_2(A_1 - A_2)\frac{1}{P}\frac{dP}{dx}\right\} \\ &A_i = \left(\frac{\pi}{\beta_i}\right)^{1/2}\frac{b_i}{2k_0 D_{12}}, \qquad B = \frac{k_0 D_{12}}{2P}\left[A_1 A_2 + \frac{A_1 y_2 \mu_1^2 + A_2 y_1 \mu_2^2}{y_1 y_2 (\mu_1+\mu_2)^2}\right] \\ &C = A_1 y_2 + A_2 y_1\end{aligned} \tag{10}$$

The knowledge of the difference between the velocities of components of the medium averaged over the cross-section, is of practical importance. In the usual hydrodynamic formulation this difference is zero. We have

$$\langle u_1\rangle - \langle u_2\rangle = -\frac{D_{12}}{y_1 y_2}\frac{dy_1}{dx} - \frac{1}{1+C}\left[\frac{1}{\mu_1+\mu_2}\left(\frac{\mu_2}{y_2} - \frac{\mu_1}{y_1}\right) + A_1 - A_2\right]\frac{D_{12}}{P}\frac{dP}{dx} \tag{11}$$

In the case of a flow in a cylindrical pipe the solution is obtained in exactly the same manner, therefore we shall give the result at once.

After averaging over the cross-section of the pipe, the solution of the initial system of equations (1) for an arbitrary value of the velocity $w_i(R)$ at the wall, has the form

$$\langle w_1\rangle = -\frac{R^2}{8(\mu_1+\mu_2)}\frac{dP}{dx} + w_1(R) - \frac{1}{k_0^2}\left[\omega_1 + \frac{d}{\mu_1}(w_1(R) - w_2(R))\right] \times \left[1 - \frac{2\zeta(z)}{z}\right] \tag{12}$$

$$z = k_0 R, \qquad \zeta(z) = \frac{I_1(z)}{I_0(z)}$$

Here $I_k(z)$ is a modified Bessel function and R is the pipe radius. We have for $w_1(R)$

$$w_1(R) = -\frac{k_0 D_{12} A_1}{P_1(1 + C\zeta(z))}\left\{\frac{\mu_1 \omega_1}{k_0}\zeta(z) + \left[\frac{\mu_1}{\mu_1+\mu_2} + A_2 y_1 \zeta(z)\right]\frac{R}{2}\frac{dP}{dx}\right\} \tag{13}$$

Expressions (12) and (13) together yield the required solution for the problem of a flow with slippage in a cylindrical pipe.

For the component velocity difference we have

$$\begin{aligned}\langle w_1\rangle - \langle w_2\rangle = &-\left[1 - \frac{2\zeta(z)}{z(1+C\zeta(z))}\right]\frac{D_{12}}{y_1 y_2}\frac{dy_1}{dx} - \\ &(1 + C\zeta(z))^{-1}\left[\frac{\mu_2 y_1 - \mu_1 y_2}{y_1 y_2(\mu_1+\mu_2)}\left(1 - \frac{2\zeta(z)}{z}\right) + \right. \\ &\left.(A_1 - A_2)\zeta(z)\right]\frac{D_{12}}{P}\frac{dP}{dx}, \qquad \zeta(z) = \frac{I_1(z)}{I_0(z)}\end{aligned} \tag{14}$$

The results simplify when $z \sim \mathrm{Kn}^{-1} \gg 1$, and the formula for $\langle w_1\rangle$ is written in the form (10) where the expression in the first set of brackets is replaced by

$$\frac{R^2}{8(\mu_1+\mu_2)} + \frac{BR}{1+C}$$

and the expression for $\langle w_1\rangle - \langle w_2\rangle$ retains the form of (11).

Comparing the results (10)—(14) with the analogous results of /1/ (the latter can be obtained from (10)—(14) by setting formally $A_i = 0$) we find, that within the framework of (1) the boundary slippage condition must, for the quantitative reasons, be taken into account. From the physical point of view this is justified by the fact that the equations (1) include in the first approximation the effect of the finite character of the Knudsen numbers on the motion of the gas mixture, with the boundary conditions of adhesion and slippage corresponding, respectively, to the zero and first (in Kn) approximation. Naturally, only the solution of (1) with slippage is correct. We find that allowing for a certain specified amount of slippage is of principal importance. Thus for the practically important quantity

$$\Delta y = y_1 y_2 (\langle w_1 \rangle - \langle w_2 \rangle)(y_1 \langle w_1 \rangle + y_2 \langle w_2 \rangle)^{-1}, \quad m_2 > m_1$$

determining the effect of the gas diffusion separation, the solution with slippage prevents the possibility of inverting this effect (change of sign) in the flows of nonisotropic mixtures. The results obtained here give a correct extrapolation to the region of free molecular flows $(\mathrm{Kn} \gg 1)$

$$\Delta y \to y_1 y_2 (A_1 - A_2)(A_1 y_1 + A_2 y_2)^{-1}$$

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